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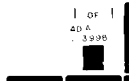

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A UNIVARIATE MONTE CARLO TECHNIQUE TO APPROXIMATE RELIABILITY CONFIDENCE LIMITS OF SYSTEMS WITH COMPONENTS CHARACTERIZED BY THE WEIBULL DISTRIBUTION

THESIS

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

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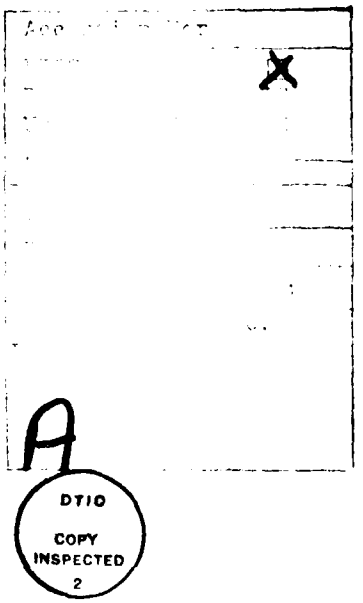

December 1979

Preface

Reliability engineering is a critical area in the design of any system. Too often a promising and exciting system which is on the threshold of technology turns into a nightmare to maintain because it is constantly breaking down. Improvements are clearly needed in the field of estimating system reliability. For several years, thesis students under the direction of Dr. Albert H. Moore have been doing research in the area of reliability, especially with regard to Monte Carlo simulation of systems reliability. This thesis explores the possibility of simplifying reliability analysis while still maintaining reasonable accuracy.

I want to thank Dr. Moore for his patient instruction over the last months and his continuing interest in this thesis. I also would like to thank my wife Nancy for the daily encouragement she has given me for the past eighteen months.

I would like to thank Phyllis Reynolds for typing my thesis.

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Abstract

A univariate Monte Carlo technique is developed for the determination of lower confidence limits of system reliability based on component test data. It is assumed that the component test data consists of failure times which are distributed according to a known two-parameter Weibull probability distribution. These failure times are randomly generated using the true shape and scale parameters of the distribution. Maximum-likelihood estimators are found for the shape and scale parameters and then substituted into the reliability equation to obtain the maximum-likelihood estimator for the component reliability. The estimated bias in this estimator is subtracted to yield an approximately unbiased estimator of the component reliability. Using the empirical variance of the reliability estimate and assuming a normal distribution, a Monte Carlo simulation is run for four hypothetical systems consisting of as many as five components. The simulation is repeated 600 times. Since the true reliability is known, on each run it can be determined if the desired confidence intervals contain the true system reliability. The result is an absolute measure of the effectiveness of the univariate technique. The entire simulation was run for component test data sample sizes ranging from ten to one hundred. A second run of 600 was made to examine the Monte Carlo sampling error for component test sample sizes of 100.

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I. Introduction

Problem Statement

Testing a complex system in order to determine its reliability can be expensive in terms of time and money. Even the preliminary design of a system requires advance knowledge about how a network of components will have to be structured to yield the desired system reliability. The purpose of this thesis is to determine system reliability based on reliability data about individual components within a system.

Reliability

The reliability of a system is defined as the probability that the system will perform its intended function for a specified period of time under stated conditions. The current importance of reliability is due in large part to the experience of the U.S. military during and immediately after World War II. Tremendous difficulties were encountered in assuring reliable equipment for the fighting

forces. For example, during the war, 50% of all stored airborne electronic equipment became unserviceable before even being used. The Army reported high truck engine and power plant mortality rates. In 1947, the Navy stated that 70% of its electronic equipment did not operate satisfactorily (Ref 12:Sec I, 1-2). Although reliability theory and techniques for reliability estimation have shown magnitudes of improvement since World War II, there is still the need for significant developments. As systems become more complex and testing becomes more expensive (look at the space shuttle as an example), accurate reliability estimation techniques based on small sample sizes must be developed.

The Weibull Distribution

The Weibull probability density function was initially developed for the study of fatigue failure in materials. Later, it was discovered that electron tubes of various types fail according to the distribution (Ref 12: Sec II, 55). Thus the Weibull can be used to express the distribution of time to failure for many mechanical and electrical devices. In addition, there is frequently the assumption that component failures (especially electronic components) are distributed according to the exponential distribution. Zelen and Dannemiller (Ref 15) have pointed out that the exponential distribution is generally not a robust approximation to the Weibull, especially if the shape parameter is greater than 1.

The Weibull density function is

$$f(t;k,\theta,c) = \frac{k(t-c)^{k-1}}{\theta^k} e^{-\left(\frac{t-c}{\theta}\right)^k} \qquad k,\theta \geq 0;\ c \leq t \quad (1)$$

$$= 0 \qquad \text{elsewhere,}$$

where t is the time, θ is the scale parameter, c is the location parameter, and k is the shape parameter. The scale parameter determines the spread or dispersion of the function about its mean. The location parameter determines the point of origin of the distribution. The shape parameter determines whether the failure rate is decreasing, increasing, or invariant with time. This gives the Weibull distribution flexibility to model components with failure rates that decrease, increase, or remain constant over time. For example, if the shape parameter is 1, the Weibull degenerates to the exponential density function. If the shape parameter is 3.5, the Weibull is an excellent approximation to the normal distribution. A shape parameter of 2 yields a function which, when correctly scaled, can approximate Beta distributions that are skewed towards the right. Fig. 1 shows the flexibility of the Weibull to model the exponential, the normal, and the Beta distributions.

An individual component that follows a Weibull distribution in time to failure will have its reliability expressed by

$$R(t) = \exp(-((t-c)/\theta)^k). \qquad (2)$$

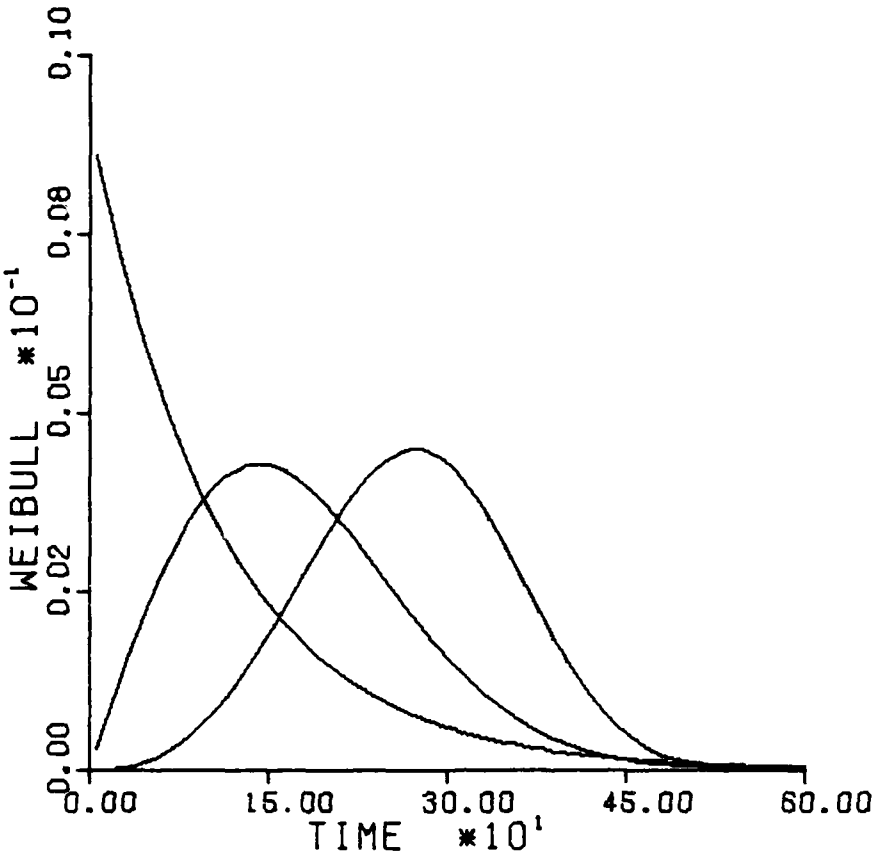

Fig. 1. Weibull Distribution with Shape Parameters of 1, 2, and 3.5

Therefore, determining the reliability of one component is a relatively easy procedure of substituting into the reliability equation. However, one is frequently interested in determining the reliability of a system of several components, each possessing a time to failure given by the Weibull probability density function. Problems arise in that analytical techniques designed to solve for the reliability of complex systems quickly become overwhelming as the number of components increases. Therefore, computer

simulation has become a handy tool to utilize on complex systems.

Survey of Past Work

Some of the original work in the use of Monte Carlo methods for determining confidence limits on system reliability was done by Donald Orkand (Ref 8) in 1960. He pointed out the serious limitations in using only point estimates of component reliability to determine a point estimate of the system reliability. He also showed the flexibility of using the Monte Carlo method and its adaptability to the circuitry of various complex systems. Albert Bernhoff (Ref 2) explained the problems of applying analytical approaches to find system reliability confidence intervals by combining component reliability confidence intervals. He concluded that the analytical approaches generally do not work. In 1967, Louis Levy and Albert Moore (Ref 5) developed a Monte Carlo technique for obtaining reliability confidence intervals for systems with components characterized by the exponential, normal, lognormal, Gamma, and Weibull distributions. Robert Lannon (Ref 4) established reliability confidence intervals for the Weibull distribution using a bivariate analysis of the shape and scale parameters. Robert Snead (Ref 11) studied reliability estimates using the Weibull, Gamma, and Logistic distributions. For each distribution, Snead used the property that the reliability estimates are asymptotically normal.

Thus he was able to perform a univariate analysis using just the reliability parameter, as long as large sample sizes existed. However, because the primary value in the Monte Carlo method is to reduce the prohibitively expensive testing of large samples of systems and their individual components, it would be extremely helpful if a univariate method could be developed for small sample sizes.

Darrel Thoman, Lee Bain, and Charles Antle (Refs 13 and 14) have done considerable research into the Weibull probability distribution and the maximum likelihood estimation of the Weibull parameters. Key results from their studies show that, assuming the two-parameter Weibull (that is, the shape and scale parameters are unknown while the location parameter is zero), the distribution of the maximum likelihood estimator of reliability, $\hat{R}(t)$, depends only on the true system reliability R(t) and the sample size n. Also, $\hat{R}(t)$ is nearly unbiased and has a variance practically equal to the Cramer-Rao lower bound for the variance of an unbiased estimator (Ref 13:363). These results form the basis for the research done in this thesis.

Objectives

There are two objectives of this study. The first is to simplify the bivariate analysis of Lannon by mapping the shape and scale parameters onto the reliability parameter, resulting in a univariate analysis. The second is to explore how far one can reduce the sample sizes of component

test data while retaining reasonably accurate confidence intervals.

Approach

Several assumptions are necessary in order to make the problem tractable. The first is that system components have failure times represented by the Weibull probability distribution. Also, all components must have been subjected to reliability tests, allowing estimation of the shape, scale, and location parameters. The distribution of the estimators for the parameters must also be known. Finally, there must be a known mathematical relationship between the system reliability and component reliabilities (Ref 7:459). In this thesis, it is assumed that the components fail independently of one another. This allows for the use of standard formulas for the reliability of systems having series or parallel connections, and for the reduction of complex systems into combinations of series and parallel configurations. Orkand (Ref 8:7-8) describes procedures to apply if there is a dependence relation among system components.

The fundamental approach that will be applied throughout this thesis is to determine reliability estimates and the associated confidence intervals using the Monte Carlo method. This method uses random sampling to investigate the solution of deterministic or stochastic problems. In essence, one inputs a set of random variables

from specified probability distributions, and then solves the problem for each set of inputs to obtain a random sample of outcomes (Ref 7:459). The steps to the method are as follows:

1. Using the component test data (set of failure times), calculate the maximum likelihood estimators for the parameters of the underlying distribution. Do this for all components.

2. Determine the joint distribution of the maximum likelihood estimators. Repeat this for all components.

3. Generate a sample value for each parameter by sampling from the distribution of the parameter.

4. Substitute the sample parameters into the component's reliability equation to derive a sample component reliability.

5. Repeat steps 3 and 4 for each component, and then substitute the sample component reliabilities into the system reliability equation to obtain a sample system reliability.

6. Repeat steps 3-5 to obtain the desired number of samples of the system reliability.

7. Order the system reliability samples and determine the desired confidence limits.

The primary advantage of the Monte Carlo method is its adaptability to any type of complex system. It is a proven method that can be used for the initial investigation into any systems reliability problem that fits the

above-mentioned assumptions. It is also a valuable method to apply when other methods fail due to the complexity of the system (Ref 7:459).

II. Theoretical Development

Step 2 of the Monte Carlo method requires knowledge about the joint distribution of the maximum likelihood estimators. Since an objective of this thesis is to research a univariate Monte Carlo method, the distribution of the single estimator, $\hat{R}(t)$, is important.

Maximum Likelihood Estimation

An estimator is simply an approximation to the unknown value of some population parameter (Ref 9:39). For example, if one is interested in the mean utility bill in some city, he can sample from several different households and use the sample mean as an estimator to the mean utility bill for all the households within the city. Also, the sample standard deviation is frequently used as an estimator for the population standard deviation. A particular type of estimator is the maximum likelihood estimator (MLE). An example can best explain this estimator. Assume that someone has a box containing ten balls, each of which is either black or white, and is asked to estimate the number of black balls based on a sample of three balls drawn from the box. If the sample is three black balls, then the maximum likelihood estimator of the total number of black balls is ten. This is because a population of ten black balls would maximize the probability of drawing the sample

of three black balls. In this case, that probability would be one. Maximum likelihood estimators are those values of the parameters which maximize the probability or the joint density (the likelihood) of the observed sample (Ref 6:302).

Some symbols and special terminology need to be introduced here. Let $y_1,y_2,\ldots,y_n$ be n sample observations taken on the corresponding random variables $Y_1,Y_2,\ldots,Y_n$. Then if $Y_1,Y_2,\ldots,Y_n$ are discrete random variables, the likelihood of the sample, L, is defined to be the joint probability of $y_1,y_2,\ldots,y_n$. If $Y_1,Y_2,\ldots,Y_n$ are continuous random variables, then the likelihood L is defined to be the joint density evaluated at $y_1,y_2,\ldots,y_n$. In either the discrete or the continuous case, maximum likelihood estimators are those values of the estimators which maximize L, the likelihood of the sample (Ref 6:303).

In researching the reliability of systems with components that fail according to the Weibull probability distribution, the sample is a set of n failure times (say t_1, $t_2,\ldots,t_n$) obtained by testing each component. The objective is to pick the values of the shape and scale parameters (location parameter is assumed to be zero) that will maximize the likelihood of the observed failure times. As defined earlier, the likelihood L equals the joint density of the sample.

$$L = f(t_1,t_2,\ldots,t_n;\ k,\theta)$$

But since the failure times are independent of one another,

$$L = f(t_1;\ k,\theta)f(t_2;\ k,\theta)\ldots f(t_n;\ k,\theta)$$

To maximize L, the usual procedure is to take the partial derivatives of L with respect to the parameters, set the derivatives equal to zero, and then solve for the values of the parameters. But maximizing ln(L) is easier than maximizing L and will result in the same estimators for the shape and scale parameters.

$$\ln(L) = \ln\left[\prod_{i=1}^{n} f(t_i;\ k,\theta)\right]$$

$$= \sum_{i=1}^{n} \ln\left[\frac{k t_i^{\,k-1}}{\theta^k}\ e^{-(\frac{t_i}{\theta})^k}\right]$$

$$= \sum_{i=1}^{n} [\ln(k) + (k-1)\ln(t_i) - k\ln(\theta) - (\frac{t_i}{\theta})^k\ ]$$

$$= n\ln k + (k-1)\sum_{i=1}^{n} \ln\ t_i - nk\ln\theta$$

$$- \theta^{-k}\sum_{i=1}^{n} t_i^{\,k} \qquad (3)$$

Now taking the partial derivative of ln(L) with respect to θ,

$$\frac{\partial \ln L}{\partial \theta} = -nk/\theta + k\theta^{-k-1} \sum_{i=1}^{n} t_i^k \qquad (4)$$

Setting Eq (4) equal to zero and solving for θ yields

$$\hat{\theta} = \left[\frac{\sum_{i=1}^{n} t_i^k}{n} \right]^{1/k} \qquad (5)$$

Substituting the value of $\hat{\theta}$ into Eq (3) and taking the partial derivative of ln(L) with respect to k,

$$\frac{\partial \ln L}{\partial k} = n/k + \sum_{i=1}^{n} \ln(t_i) - n\left[\sum_{i=1}^{n} t_i^k \ln(t_i)\right] / \sum_{i=1}^{n} t_i^k \qquad (6)$$

Setting the derivative equal to zero yields

$$1/\hat{k} - \frac{\sum_{i=1}^{n} t_i^{\hat{k}} \ln t_i}{\sum_{i=1}^{n} t_i^{\hat{k}}} + \frac{1}{n} \sum_{i=1}^{n} \ln t_i = 0 \qquad (7)$$

Eqs (5) and (7) are two equations in the two unknowns, $\hat{k}$ and $\hat{\theta}$. Simultaneous solution of these equations yields the maximum likelihood estimators of k and θ.

The Distribution of $\hat{R}(t)$

The MLE of reliability is found by substituting the MLEs for k and θ into the reliability formula, Eq (2).

$$\hat{R}(t) = e^{-(\frac{t}{\hat{\theta}})^{\hat{k}}} \tag{8}$$

If one repeatedly draws samples of size n from a population distributed according to the Weibull, and uses the previously mentioned technique to find the MLE of reliability for each sample, then a distribution of the MLE $\hat{R}(t)$ will result. Let $\hat{\theta}_s = (\hat{\theta}/\theta)^k$ and $\hat{k}_s = \hat{k}/k$. Thoman, Bain, and Antle (Ref 13) have proven that the joint distribution of $\hat{\theta}_s$ and $\hat{k}_s$ is independent of θ and k. They go on to prove (Ref 14) that the distribution of $\hat{R}(t)$ depends only on R(t).

$$\ln \hat{R}(t) = -(t/\hat{\theta})^{\hat{k}}$$

$$\ln[-\ln \hat{R}(t)] = \hat{k} \ln(t/\hat{\theta})$$

$$= \hat{k}/k \ln[(t/\hat{\theta})^k]$$

$$\ln[-\ln(\hat{R}(t))] = \hat{k}/k \ln[(t/\theta)^k (\hat{\theta}/\theta)^{-k}]$$

$$= \hat{k}_s \ln[(-\ln R(t))\hat{\theta}_s^{-1}]$$

Since $\hat{k}_s$ and $\hat{\theta}_s$ are distributed independently of k and θ, the distribution of $\hat{R}(t)$ depends only on R(t). Thus the parameters t, k, and θ can affect the distribution or $\hat{R}(t)$ only through R(t) (Ref 14:363). This is a significant result. It allows for the univariate analysis of R(t), in place of the bivariate analysis (using k and Θ) that Lannon did. What is being said is that, if two components have different underlying Weibull distributions (different shape and scale parameters), but both have the same

reliability, then they will have identical distributions of their maximum likelihood estimators of reliability.

An important criteria of an estimator is its bias. Usually a small or zero bias is desired. The bias, B, is defined as the expected value of the estimator minus the true value of the parameter being estimated. In this case,

$$B = E[\hat{R}(t)] - R(t)$$

The bias of $\hat{R}(t)$ can be estimated by Monte Carlo simulation. Thoman, Bain, and Antle (Ref 14:365) derived 10,000 estimates of R(t) using sample sizes of 8 to 100 and true reliabilities of 0.5 to 0.98. The 10,000 estimates were averaged to obtain $E[\hat{R}(t)]$, and then the true reliability was subtracted to yield the bias. Unfortunately, Thoman, Bain, and Antle did not include enough significant digits to be helpful. David Antoon (Ref 1) did a Monte Carlo analysis and derived 2000 estimates of R(t). The bias was calculated for the same range of sample sizes and true reliabilities as Thoman, Bain, and Antle. Antoon's results compared favorably and are shown in Table I. As can be seen, the bias of $\hat{R}(t)$ is very close to zero. Thus the MLE, $\hat{R}(t)$, is almost an unbiased estimator of R(t).

Another desired characteristic of an estimator is that it have a small variance. In this case,

$$Var[\hat{R}(t)] = \sum_{i=1}^{M} \frac{[E(\hat{R}(t))-R(t)]^2}{M-1}$$

TABLE I

BIAS OF $\hat{R}(t)$ (Ref 1)

R(t)	Sample Size 10	12	15	20	25	30	40	50	75	100
.500	.00494	.00577	.00405	.00317	.00197	.00097	.00100	.00109	.00071	.00017
.550	.00944	.00976	.00610	.00519	.00388	.00307	.00367	.00310	.00275	.00203
.600	.00749	.00692	.00588	.00345	.00206	.00283	.00292	.00251	.00131	.00021
.650	.01232	.01088	.00806	.00632	.00499	.00369	.00454	.00323	.00261	.00210
.700	.01034	.00581	.00585	.00506	.00432	.00373	.00308	.00204	.00101	.00107
.750	.00881	.00821	.00635	.00459	.00386	.00432	.00343	.00213	.00184	.00054
.800	.01156	.01092	.00941	.00728	.00600	.00538	.00403	.00333	.00148	.00036
.850	.00530	.00493	.00483	.00453	.00262	.00223	.00223	.00114	.00158	.00149
.900	.00329	.00257	.00103	.00084	.00095	.00125	.00165	.00116	.00105	.00079
.925	.00304	.00200	.00208	.00103	.00089	.00004	.00044	-.00004	-.00035	-.00012
.950	.00062	.00039	-.00026	.00075	.00070	.00018	.00010	.00015	.00004	.00028
.980	-.00232	-.00192	-.00138	-.00118	-.00088	-.00075	-.00047	-.00029	-.00036	-.00027

where M is the Monte Carlo size, or the number of samples generated. As mentioned, Thoman, Bain, and Antle generated 10,000 samples from which they calculated estimates of the variance of $\hat{R}(t)$. Table II (Ref 14:366) shows their results. As can be seen, only three significant digits accuracy is reported. Antoon (Ref 1) calculated the empirical variance of $\hat{R}(t)$ based on 2000 samples and the results in Table III show empirical variance as a function of sample size and reliability.

It is useful to compare the empirical variance of $\hat{R}(t)$ with the Cramer-Rao Lower Bound (CRLB). The CRLB describes the lower limit for the variance of an unbiased estimator. Although $\hat{R}(t)$ is not unbiased, Table I does show the bias approaching zero as the sample size increases. Thus the asymptotic variance of $\hat{R}(t)$ equals the CRLB. Rao's results on the asymptotic distribution of a function of asymptotic normal variables (Ref 10) can be applied to $\hat{R}(t)$.

$$\text{CRLB} = \sigma^2_{\hat{\theta}} \left[\frac{\partial R}{\partial \theta}\right]^2 + 2\sigma_{\hat{\theta},\hat{k}} \frac{\partial R}{\partial \theta} \frac{\partial R}{\partial K} + \sigma^2_{\hat{k}} \left[\frac{\partial R}{\partial K}\right]^2$$

where $\sigma^2_{\hat{\theta}}$, $\sigma_{\hat{\theta},\hat{k}}$, and $\sigma^2_{\hat{k}}$ are elements of the asymptotic covariance matrix of $(\hat{\theta},\hat{k})$. Thoman, Bain, and Antle (Ref 13:449) show that

$$\sigma^2_{\hat{\theta}} = 1.109\theta^2/(nk^2) \quad \sigma^2_{\hat{k}} = .608k^2/n \quad \sigma_{\hat{\theta},\hat{k}} = .257\theta/n$$

TABLE II

VARIANCE OF $\hat{R}(t)$ (Ref 14)
(X 10000)

R(t)	Sample Size 10	12	15	20	25	30	40	50	75	100
.500	200	167	124	90	72	59	43	34	23	17
.600	187	154	118	86	68	57	42	33	22	16
.700	153	126	99	72	58	48	36	29	19	14
.750	130	107	86	62	50	42	31	25	17	12
.800	103	86	70	51	41	34	26	20	14	10
.850	76	63	51	37	30	25	19	15	10	8
.900	47	39	32	23	19	16	12	9	6	5
.925	33	27	22	16	13	11	8	7	4	3
.950	19	16	13	9	7	6	5	4	3	2
.980	5	4	3	2	2	1	1	1	1	1

TABLE III

VARIANCE OF $\hat{R}(t)$ (Ref 1)
(X 10000)

R(t)	Sample Size 10	12	15	20	25	30	40	50	75	100
.500	189.999	157.227	123.899	93.065	69.722	58.308	43.930	34.987	22.420	16.500
.550	188.788	157.904	117.137	88.078	67.684	57.078	40.056	32.925	22.382	16.736
.600	180.177	147.234	119.377	87.535	70.124	58.354	39.841	32.183	22.973	16.876
.650	164.635	136.539	108.973	81.433	64.272	52.563	39.100	30.769	20.376	16.152
.700	147.987	123.588	97.674	72.863	55.681	45.901	35.023	30.184	18.156	14.592
.750	122.877	102.333	82.774	63.155	49.210	41.628	30.780	24.197	16.120	12.525
.800	96.118	76.755	62.711	48.734	39.163	32.047	23.281	19.334	13.720	9.998
.850	77.018	64.240	52.984	39.652	30.880	26.000	18.879	15.218	10.125	7.426
.900	46.731	39.829	33.074	22.458	19.149	15.785	11.540	9.327	6.325	4.902
.925	30.625	26.173	20.557	16.330	13.170	10.660	8.088	6.441	4.516	3.404
.950	18.784	15.745	13.220	9.660	7.779	6.698	4.893	3.980	2.634	1.814
.980	6.145	4.933	3.640	2.602	1.971	1.674	1.230	.972	.651	.491

Therefore (Ref 14:365),

$$CRLB = R^2 (\ln R)^2 [1.109 - .514 \ln(-\ln R) + .608[\ln(-\ln R)]^2]/n \quad (9)$$

One can see that the CRLB approaches zero as the sample size gets larger or the reliability approaches one. Snead (Ref 11:22) found the CRLB by using the asymptotic covariance matrix and numerous algebraic operations involving n, t, $\hat{k}$, and $\hat{\theta}$. Eq (9) represents the CRLB as a function of just R(t) and n. This is a significant simplification over Snead's method without the loss of any accuracy. Eq (9) was used to generate the data in Table IV which shows the CRLB as a function of sample size and reliability. One would expect that the Table IV values should be less than the corresponding empirical variances in Table III. This is generally correct, especially at low reliabilities and small sample sizes. Occasionally, due to Monte Carlo variability, the empirical variance is less than the CRLB.

TABLE IV

CRAMER-RAO LOWER BOUND
(X 10000)

R(t)	Sample Size 10	12	15	20	25	30	40	50	75	100
.500	165.644	138.036	110.429	82.822	66.257	55.215	41.411	33.129	22.086	16.564
.550	165.885	138.238	110.590	82.943	66.354	55.295	41.471	33.177	22.118	16.589
.600	162.384	135.320	108.256	81.192	64.954	54.128	40.596	32.477	21.651	16.238
.650	154.699	128.915	103.132	77.349	61.879	51.566	38.675	30.940	20.626	15.470
.700	142.444	118.704	94.963	71.222	56.978	47.481	35.611	28.489	18.993	14.244
.750	125.375	104.479	83.583	62.688	50.150	41.792	31.344	25.075	16.717	12.538
.800	103.501	86.251	69.001	51.751	41.401	34.500	25.875	20.700	13.800	10.350
.850	77.289	64.407	51.526	38.644	30.915	25.763	19.322	15.458	10.305	7.729
.900	48.058	40.048	32.039	24.029	19.223	16.019	12.014	9.612	6.408	4.806
.925	33.173	27.644	22.115	16.586	13.269	11.058	8.293	6.635	4.423	3.317
.950	18.995	15.829	12.663	9.497	7.598	6.332	4.749	3.799	2.533	1.899
.980	4.849	4.041	3.233	2.425	1.940	1.616	1.212	.970	.647	.485

III. Procedure

The essential thrust of this thesis is to test a univariate method of finding confidence intervals for the reliability of complex systems over a specified mission time. The method will incorporate the knowledge gained in Chapter II pertaining to the bias and empirical variance of $\hat{R}(t)$.

Components

To fully test the method, a variety of systems composed of a variety of components should be considered. The true reliability of component i, R_i, is found by substituting into the Weibull reliability formula, Eq (2). In each case, the mission time, t, is arbitrarily set at 100 hours. The following components were used:

Component 1
- Failure Distribution: Weibull
- Parameter Values: $k = 2 \quad \theta = 250 \quad c = 0$
- True Reliability: $R_1 = \exp[-(100/250)^2] = .85214$

Component 2
- Failure Distribution: Weibull
- Parameter Values: $k = 3 \quad \theta = 210 \quad c = 0$
- True Reliability: $R_2 = \exp[-(100/210)^3] = .89765$

Component 3
- Failure Distribution: Weibull
- Parameter Values: $k = 2 \quad \theta = 300 \quad c = 0$
- True Reliability: $R_3 = \exp[-(100/300)^2] = .89484$

Component 4
  Failure Distribution Weibull
  Parameter Values $k = 3.5 \quad \theta = 150 \quad c = 0$
  True Reliability $R_4 = \exp[-(100/150)^{3.5}] = .78512$

Component 5
  Failure Distribution Weibull
  Parameter Values $k = 2.5 \quad \theta = 250 \quad c = 0$
  True Reliability $R_5 = \exp[-(100/250)^{2.5}] = .90376$

System Networks

The five components were combined to form various kinds of systems. Looking at a variety of systems helps to assure that the results are not unique to the particular configuration of a system. Four different systems were used and are shown in Fig. 2.

System 1 consists of three components in series. The reliability of a system with components connected in series is found by taking the products of the individual component reliabilities. Therefore, the true reliability of System 1, R_{s_1}, is

$$R_{s_1} = R_1 R_2 R_3 \tag{10}$$

$$= (.85214)(.89765)(.89484)$$

$$= .68448$$

System 2 consists of one component connected in series with two in parallel. To simplify the reliability equations, let Q_i be the probability of failure for component i. Therefore, $Q_i = 1-R_i$. The true reliability of

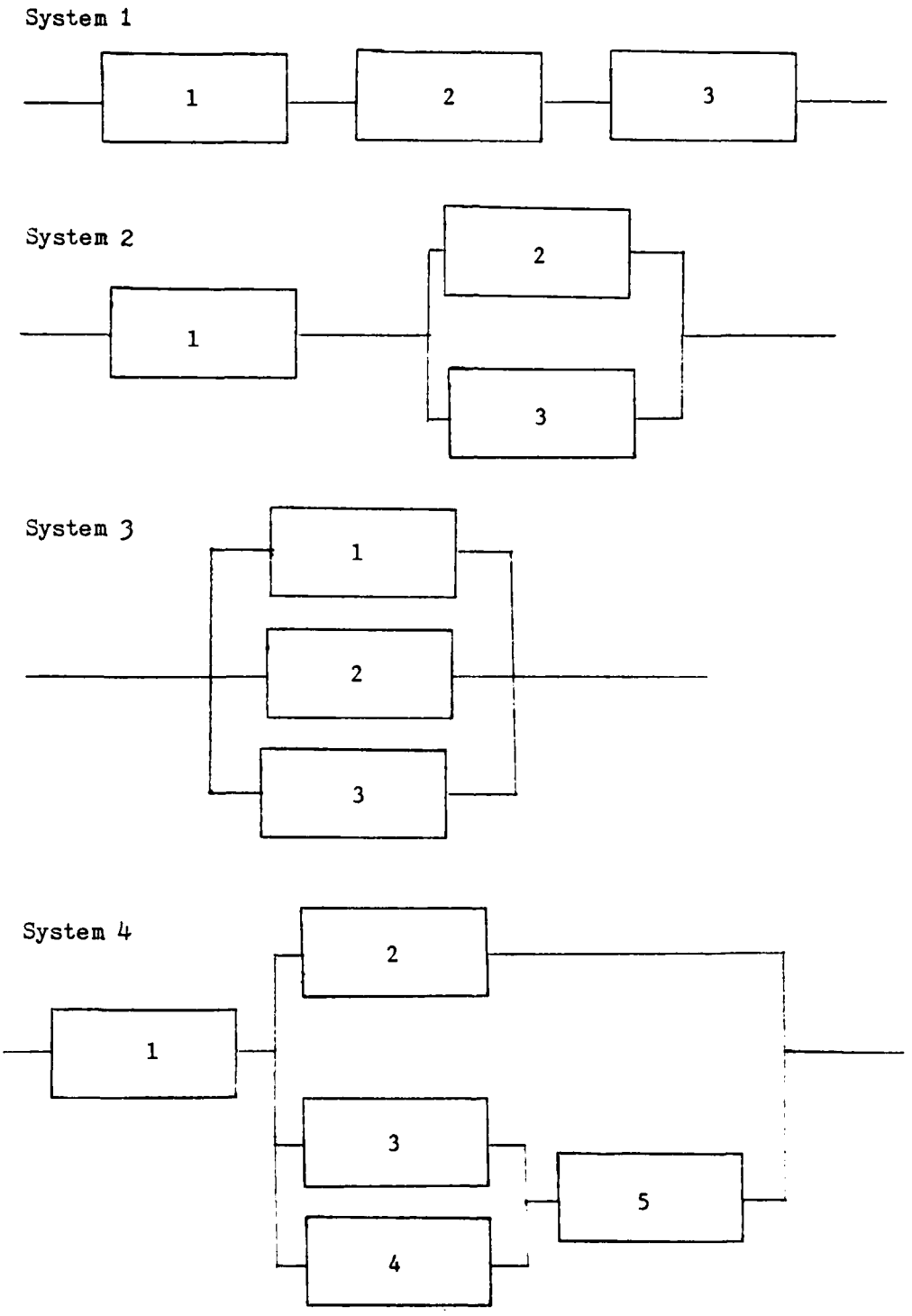

Fig. 2. Systems 1, 2, 3, and 4

System 2, R_{s_2}, is

$$R_{s_2} = R_1(1 - Q_2Q_3) \tag{11}$$

$$= .85214[1 - (.10235)(.10516)]$$

$$= .84297$$

System 3 consists of three components connected in parallel. The reliability of this system is simply one minus the probability that all components fail. Thus the true reliability of System 3, R_{s_3}, is

$$R_{s_3} = 1 - Q_1Q_2Q_3 \tag{12}$$

$$= 1 - (.14786)(.10235)(.10516)$$

$$= .99841$$

All systems can be seen as consisting of subsystems of components which are either series-connected (such as System 1) or parallel-connected (such as System 3). The subsystems are in turn connected to one another in either series or parallel. Thus one can combine subsystem reliabilities using either the series formula, Eq (10), or the parallel formula, Eq (12), until the system reliability is found. This was done for System 2 and will now be done for the larger complex network, System 4.

$$R_{s_4} = R_1[1 - Q_2[1 - R_s(1 - Q_3Q_4)]] \quad (13)$$

$$= .85214[1 - .10235[1 - .90376(1 - (.10516)(.21488))]]$$

$$= .84197$$

The Monte Carlo Method

Now the true reliabilities of all five components and all four systems have been calculated. But, if the reliability engineer had all the true reliabilities, he would not need to go through any kind of Monte Carlo simulation. In this study, the true reliabilities are considered because, after the simulation is over, the true reliabilities will provide an absolute measure against which the univariate method can be gauged.

To find confidence intervals about the true system reliabilities, it is desired to modify the Monte Carlo method presented in Chapter I to incorporate the theoretical knowledge gained in Chapter II about the bias and empirical variance of $\hat{R}(t)$. The steps to the Monte Carlo method as applied in this thesis are as follows. A discussion of each step follows this list.

1. Using the true shape, scale, and location parameters, generate a simulated sample of test data (component failure times) from the Weibull distribution.

2. Based on the simulated test data, calculate the maximum likelihood estimators of the shape and scale

parameters. The location parameter is assumed to be zero.

3. Substitute the estimators of the shape and scale parameters into the reliability equation to obtain a maximum likelihood estimator of the component reliability.

4. Subtract the bias from the maximum likelihood estimator of the reliability to obtain an unbiased estimator of the component reliability.

5. Form a vector of component reliability estimates distributed normally about the unbiased estimator.

6. Repeat steps 1-5 for each component.

7. Using the reliability equation of a complex system, combine the vectors of component reliabilities to obtain a vector of system reliabilities.

8. Order the vector of system reliabilities and determine the 99, 95, 90, 80, 70, 60, and 50 percent one-sided confidence intervals. Note if each of these intervals contains the true system reliability.

9. Repeat steps 1-7 until the desired Monte Carlo size is reached.

10. To measure the accuracy of the confidence limits, determine the percentage of the runs in which each of the confidence intervals covered the true system reliability.

The computer program which executes the above Monte Carlo method is shown in the appendix. Some elaboration is necessary to explain individual steps of the technique.

In step 1, Weibull-distributed sample failure times were generated by using the subroutine GGWIB from the International Mathematical and Statistical Libraries (IMSL). In step 2, the MLEs of the shape and scale parameters ($\hat{k}$ and $\hat{\theta}$) were determined by simultaneous solution of Eq (5) and Eq (7) using the nonlinear simultaneous equation solver ZSYSTM, also from the IMSL library. In step 3, the MLE of reliability, $\hat{R}(t)$, was found by substitution of $\hat{k}$ and $\hat{\Theta}$ into Eq (8). In step 4, the bias of the MLE $\hat{R}(t)$ was determined by interpolation (using cubic splines because of nonlinearity) in Table I. Note that the result of step 4 is an estimator that is usually less than the maximum-likelihood estimator (because the bias is usually positive). This will result in more accurate and generally smaller component reliability estimates and therefore generally smaller estimated system reliabilities. Step 4 is an attempt to improve on the optimistic (high) system reliability confidence limits generated by Snead (Ref 11) in his univariate asymptotic method. Step 5 is a crucial step. Here the assumption was made that the unbiased component reliability estimates are distributed normally. The mean is assumed to be the unbiased $\hat{R}(t)$ obtained from step 4. The variance is found by interpolating out of Table III, again using a cubic splines interpolation because of nonlinearity in the table. Since this interpolated variance should be no less than the Cramer-Rao Lower Bound, the CRLB is also calculated using Eq (9).

Then the higher of the two is used as the variance to form a vector of 2000 random and normally-distributed component reliability estimates. (The IMSL subroutine GGNML was used to generate random normal deviates.) In step 7, the vectors of component reliability estimates are combined using the systems reliability equations [Eqs (10), (11), (12), and 13)]. The result is a vector of 2000 estimated system reliabilities for each system. These vectors are each ordered in ascending sequence and then the 1, 5, 10, 20, 30, 40, and 50 percentiles are chosen to determine the 99, 95, 90, 80, 70, 60, and 50 percent lower confidence limits. Each of these lower confidence limits is compared to the true system reliability. If the lower confidence limit is less than or equal to the true system reliability, then the associated confidence interval contains the true system reliability.

The entire experiment is repeated again and again (step 9) until the desired Monte Carlo size is reached. In this study, 600 runs of the simulation were made. In each run, and for each system and each confidence interval, it was noted if the confidence interval contained the true system reliability. Ideally, the X percent confidence interval should contain the true system reliability X percent of the time. For example, it is desired that the 95 percent confidence interval contain the true system reliability during 95 percent of the 600 runs. This provides

an absolute test (step 10) of the accuracy of this univariate method.

Additional Testing and Sensitivity Analysis

In an attempt to validate the results of the Monte Carlo technique, some additional testing was conducted. One test was to check the assumption of normality made in step 5. It is already known that estimates of component reliability are only asymptotically normal. A Kolmogorov-Smirnoff test was run to check the distribution of component reliability estimates.

Even though Table II, the variances of $\hat{R}(t)$ given by Thoman, Bain, and Antle (Ref 14:366), contains only three significant digits at most, it was decided to check if the results were sensitive to the use of that table. Also, since a faulty random number generator can significantly affect the results of any simultation, a random normal generator other than GGNML was tried. GGNML uses the inverse normal function of uniform deviates. The Box-Muller (Ref 3:610-11) was also utilized. This technique uses the natural logarithm and trigonometric functions to generate normal deviates from random uniform variables. Since GGNML and Box-Muller use different uniform deviate generators, the generation of uniform deviates is also being checked by two methods. Another desirable

characteristic of a simulation is that it not be sensitive to the choice of the initial random number seed. Therefore, simulations were run with different seeds.

IV. Results

Base Case

Tables V through VIII contain the base case results of the Monte Carlo simulation for Systems 1 through 4. For each system, the simulation was run for component test data sample sizes of 10, 15, 20, 50, and 100. Each table entry is the percentage of the Monte Carlo runs in which the simulated confidence interval covered the true system reliability. For example, in the first row of Table V and with a sample size of 10 failure times for each component, the 99 percent confidence interval contained the true reliability of System 1 only .9383 of the runs. As the component sample size increases in that first row, one can see that the confidence interval coverage improves until, at a sample size of 100, the coverage is .9817, almost 99 percent. This general improvement is also the case when looking at increasing sample sizes for the other confidence intervals.

Tables VI through VIII reveal that Systems 2, 3, and 4 experience the same improvements as component sample sizes get larger. For each system, the confidence interval coverage is low for small sample sizes. The reason for this is high or optimistic system reliability estimates. As a result, the lower confidence limits are also too high and

TABLE V

SYSTEM 1 CONFIDENCE INTERVAL COVERAGE OF THE TRUE SYSTEM RELIABILITY

Confidence Interval (Percent)	Sample Size 10	15	20	50	100
99	.9383	.9450	.9617	.9733	.9817
95	.8867	.8967	.9050	.9267	.9333
90	.8267	.8633	.8617	.8783	.8950
80	.7417	.7683	.7800	.7983	.8233
70	.6650	.6683	.6817	.7183	.7200
60	.5750	.5783	.5700	.6167	.6367
50	.4917	.5083	.4833	.5067	.5333

TABLE VI

SYSTEM 2 CONFIDENCE INTERVAL COVERAGE OF THE TRUE SYSTEM RELIABILITY

Confidence Interval (Percent)	Sample Size 10	15	20	50	100
99	.8717	.9100	.9383	.9450	.9667
95	.7933	.8467	.8600	.9083	.9200
90	.7467	.7917	.8083	.8617	.8817
80	.6750	.7017	.7200	.7767	.7917
70	.6050	.6033	.6467	.6750	.7117
60	.5083	.5067	.5433	.5567	.5950
50	.4283	.4283	.4700	.4583	.5067

TABLE VII

SYSTEM 3 CONFIDENCE INTERVAL COVERAGE OF THE TRUE SYSTEM RELIABILITY

Confidence Interval (Percent)	Sample Size 10	15	20	50	100
99	.7967	.8717	.9017	.9467	.9717
95	.6517	.7867	.7950	.8700	.9017
90	.5917	.7033	.7050	.8100	.8467
80	.4633	.5283	.5733	.6850	.7467
70	.3600	.4183	.4617	.5783	.6133
60	.2667	.3200	.3650	.4683	.5200
50	.1850	.2267	.2767	.3683	.4550

TABLE VIII

SYSTEM 4 CONFIDENCE INTERVAL COVERAGE OF THE TRUE SYSTEM RELIABILITY

Confidence Interval (Percent)	Sample Size 10	15	20	50	100
99	.8750	.9100	.9417	.9483	.9750
95	.7983	.8433	.8567	.9067	.9200
90	.7400	.7933	.8033	.8583	.8817
80	.6783	.7067	.7183	.7733	.7767
70	.5967	.6100	.6450	.6633	.7067
60	.5100	.5083	.5533	.5583	.5983
50	.4350	.4233	.4733	.4550	.4950

the confidence interval is not wide enough to cover the true system reliability.

An unexpected result from the simulation was the effect of true system reliability on the confidence interval coverage. It appears that a lower system reliability corresponds to a more accurate confidence interval. System 1, with a reliability of .68448, has consistently more accurate confidence interval coverage than any other system. Systems 2 and 4 both have reliabilities of about .84 and corresponding entries in Table VI and VIII are very close. System 3 has the highest reliability, .99841, and also the least accurate confidence intervals. Even at a sample size of 100, Table VII shows the 60 and 70 percent confidence interval coverage being eight to nine percent too low. Other systems have confidence intervals for a sample size of 100 that are always less than four percent away from the ideal accuracy.

Sensitivity Analysis

Sensitivity analysis was conducted to test some of the model inputs. To build confidence in the results of a simulation, it is desired that the results not be sensitive to the choice of random number streams. Therefore, a different initial random number seed was chosen and the results for all four systems are shown in Table IX. The 95, 90, and 80 percent confidence intervals were calculated

TABLE IX

CONFIDENCE INTERVAL COVERAGE WITH A DIFFERENT RANDOM NUMBER STREAM

Confidence Interval (Percent)	Sample Size 10	15	20	50	100
System 1					
95	.8917	.8883	.9217	.9233	.9467
90	.8350	.8317	.8917	.8717	.8850
80	.7433	.7417	.7833	.7783	.7850
System 2					
95	.8300	.8533	.8500	.8900	.9300
90	.7800	.8050	.8017	.8417	.8650
80	.6883	.6917	.7067	.7500	.7750
System 3					
95	.6750	.7433	.8133	.8550	.9050
90	.6050	.6683	.7250	.7867	.8300
80	.4650	.5200	.5833	.6667	.7150
System 4					
95	.8233	.8583	.8517	.8867	.9233
90	.7817	.8067	.7983	.8383	.8683
80	.6867	.7017	.7050	.7533	.7717

and the table reveals only slight variations from the corresponding entries in Tables V through VIII.

A faulty random number generator, especially one that does not generate enough extreme values of the distribution, can significantly affect the results of this simulation. Thus the IMSL random normal generator GGNML was replaced with the Box-Muller normal generator. The Box-Muller method is exact. That is, if the uniform generator is truly generating random uniform deviates, then the Box-Muller equations (Ref 3:610) will output truly random normal deviates. This will guarantee a proper distribution in the tails of the normal density function. Simulation runs at sample sizes of 10 and 15 were made and confidence interval coverage is shown in Table X. Again, only small variations (due to Monte Carlo variability) exist between this table and Tables V through VIII.

Thoman, Bain, and Antle calculated the empirical variances displayed in Table II. Although they reported only three significant digits, they did use a Monte Carlo size of 10,000 as opposed to Table III, in which more significant digits are shown, but the Monte Carlo size was 2000. Thoman, Bain, and Antle's table was used to see if significant variations would occur. The 95, 90, and 80 percent confidence interval coverage is shown in Table XI for sample sizes of 10, 15, 20, and 50. Differences between these results and the base case results are minor.

TABLE X

CONFIDENCE INTERVAL COVERAGE WITH THE BOX-MULLER RANDOM NUMBER GENERATOR

Confidence Interval (Percent)	Sample Size 10	15	Confidence Interval (Percent)	Sample Size 10	15
System 1			System 3		
99	.9433	.9467	99	.7950	.8667
95	.8783	.8933	95	.6733	.7367
90	.8217	.8300	90	.5900	.6650
80	.7367	.7433	80	.4417	.5117
70	.6517	.6533	70	.3667	.4250
60	.5533	.5733	60	.2783	.3033
50	.4850	.4750	50	.1883	.2183
System 2			System 4		
99	.8650	.9050	99	.8583	.9083
95	.8150	.8333	95	.8167	.8383
90	.7533	.7750	90	.7633	.7767
80	.6867	.7000	80	.6900	.6967
70	.6017	.5900	70	.5950	.5967
60	.5017	.5267	60	.5000	.5183
50	.4100	.4283	50	.4033	.4367

TABLE XI

CONFIDENCE INTERVAL COVERAGE WITH EMPIRICAL VARIANCES OF THOMAN, BAIN, AND ANTLE

Confidence Interval (Percent)	Sample Size 10	15	20	50
System 1				
95	.8917	.8817	.9217	.9233
90	.8350	.8300	.8900	.8750
80	.7417	.7400	.7833	.7800
System 2				
95	.8300	.8533	.8500	.8900
90	.7800	.8050	.8017	.8417
80	.6883	.6867	.7067	.7500
System 3				
95	.6750	.7417	.8133	.8583
90	.6050	.6650	.7250	.7867
80	.4650	.5200	.5817	.6667
System 4				
95	.8233	.8583	.8517	.8867
90	.7817	.8083	.7983	.8383
80	.6867	.6950	.7050	.7533

The sensitivity analysis has revealed only small variations (a maximum of less than five percent) between the base case and choice of random number stream, type of normal and uniform random number generator, and choice of empirical variance table. These slight differences can occur simply as a result of the variability of Monte Carlo simulation (as in any random process). A more fundamental question is the assumption of normality made in step 5 of the Monte Carlo method.

Kolmogorov-Smirnoff Test of Normality

Since the Monte Carlo size was 600, there were also 600 samples of failure times drawn for each of the five components. In turn, the sample sizes were either 10, 15, 20, 50, or 100. As a result, for each component and for each of the sample sizes, there were 600 estimates of the component reliability. A Kolmogorov-Smirnoff test was run for the distribution of these reliability estimates. The null hypothesis was that the estimates are normally distributed with a mean equal to the true component reliability and a variance equal to the interpolated value from Table III or the Cramer-Rao Lower Bound (whichever is larger). The Kolmogorov-Smirnoff test was made with a .05 probability of Type I error. That is, if the distribution is actually from the normal density function, the test still has a .05 probability of rejecting the null hypothesis.

Table XII contains the results of the test as well as the bias of the estimated reliability and the difference between the sample standard deviation and the assumed population standard deviation. There is a tendency to reject normality for sample sizes of 20 or less and to accept normality for sample sizes of 50 or greater. The table also suggests that the reason for rejecting normality is skewness in the distribution of $\hat{R}(t)$ for small sample sizes. For example, Component 3 was rejected at a sample size of 15 (with a bias of .00132 and a difference of standard deviations of -.00011), while Component 2 passed the normality test at a sample size of 100 (with a bias of -.00165 and a difference in standard deviations of .00139). If the distribution of $\hat{R}(t)$ is skewed for small sample sizes, rejection of normality would still occur even though the bias and difference in standard deviations is so small. There is a reasonable explanation for this skewness. Since the reliability estimates are bounded by zero and one, as the true component reliability approaches one, the distribution of the reliability estimates is significantly affected by the upper bound of one, but not by the lower bound of zero. Therefore, the distribution would not be symmetric, but be skewed to the left.

TABLE XII

KOLMOGOROV-SMIRNOFF TEST OF NORMALITY FOR DISTRIBUTION OF COMPONENT RELIABILITY ESTIMATES

	Sample Size				
	10	15	20	50	100
Component 1					
Reject/Pass	Reject	Reject	Reject	Pass	Pass
$E(\hat{R}) - R$	.00276	.00179	.00072	.00031	-.00003
$(s - \sigma)$	.00439	.00133	-.00034	.00149	-.00016
Component 2					
Reject/Pass	Reject	Reject	Reject	Pass	Pass
$E(\hat{R}) - T$	.00199	-.00227	-.00131	-.00073	-.00165
$(s - \sigma)$	-.00120	.00303	.00221	.00072	.00139
Component 3					
Reject/Pass	Reject	Reject	Reject	Pass	Pass
$E(\hat{R}) - R$	.00296	.00132	.00077	-.00117	-.00086
$(s - \sigma)$	-.00118	-.00011	.00209	-.00137	.00018
Component 4					
Reject/Pass	Reject	Reject	Pass	Pass	Pass
$E(\hat{R}) - R$	.00339	.00834	.00278	-.00284	.00087
$(s - \sigma)$	.00068	-.00266	-.00352	-.00024	-.00045
Component 5					
Reject/Pass	Reject	Reject	Reject	Pass	Reject
$E(\hat{R}) - R$	.00121	-.00137	.00239	-.00031	.00120
$(s - \sigma)$	.00044	.00237	.00135	.00011	-.00019

V. Conclusions and Recommendations

Conclusions

The univariate method developed in this thesis works best when component reliabilities and system reliabilities are not high (say, less than .9). If component reliabilities are higher than .9, then the skewness of the distribution of $\hat{R}(t)$ has a definite impact on the results. If the system reliability is greater than .9, then it is very sensitive to even slight changes in component reliability estimates. Therefore, even a slight skewness will significantly affect the system reliability estimates. If high component or system reliabilities exist, then they can be compensated for by large sample sizes. A sample size of 50 or greater is adequate unless the component or system reliabilities exist, then they can be compensated for by large sample sizes. A sample size of 50 or greater is adequate unless the component or system reliabilities are extremely close to one.

Recommendations

The most immediate suggestion for further work is that investigation be conducted on the distribution of component reliability estimates, $\hat{R}(t)$, based on sample sizes of less than 50. The distribution is definitely not normal.

Therefore, it would be helpful to try fitting other distributions to the empirical distribution of $\hat{R}(t)$. Since $\hat{R}(t)$ is bounded between zero and one, a logical choice is the Beta distribution. But most importantly, the Beta can also fit the skewness of the empirical distribution. Because many critical components and systems are built to demonstrate extremely high reliabilities, particular emphasis should be placed on component reliabilities of greater than .9.

Once a close fit is found for the distribution of component reliability estimates, the next step is Monte Carlo simulation of the reliability of entire systems. The computer program shown in the appendix has the capability to execute the systems simulation. Only slight modifications are required to change the sampling distribution (say the Beta instead of the normal), component test data sample size, true component reliabilities, or the configuration of systems.

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Appendix

Computer Program Listing

```
      PROGRAM  WEIB (TAPE1, INPUT, OUTPUT)
C
C     ********************************************************************
C     *                                                                  *
C     * FOR EACH COMPONENT OF A COMPLEX SYSTEM, THIS PROGRAM             *
C     * GENERATES A SAMPLE OF SIZE NSAM FROM THE WEIBULL DISTRIBUTION    *
C     * USING THE TRUE PARAMETERS TK (SHAPE), TTHETA (SCALE), AND TC     *
C     * (LOCATION).  FROM THIS SAMPLE, THE MAXIMUM LIKELIHOOD            *
C     * ESTIMATORS (MLES) FOR K AND THETA ARE DERIVED USING HARTER &     *
C     * MOORES ITERATIVE SCHEME AND THE SIMULTANEOUS EQUATION SOLVER     *
C     * ZSYSTM.  THE MLES ARE COMBINED TO YIELD RCHAT, THE MLE FOR THE   *
C     * COMPONENT RELIABILITY.  GIVEN THE RELIABILITY AND THE SAMPLE     *
C     * SIZE NSAM, RCHAT IS ASYMPTOTICALLY NORMALLY DISTRIBUTED WITH A   *
C     * SPECIFIED VARIANCE.  THEREFORE WE CAN SAMPLE FROM THE NORMAL     *
C     * DISTRIBUTION TO OBTAIN A VECTOR OF SAMPLE COMPONENT              *
C     * RELIABILITIES.  THIS PROCESS IS REPEATED FOR EACH COMPONENT      *
C     * AND THEN THE RELIABILITIES ARE COMBINED FOR 4 DIFFERENT TYPES    *
C     * OF SYSTEMS TO YIELD 4 VECTORS OF SAMPLE SYSTEM RELIABILITES.     *
C     * THESE VECTORS ARE ORDERED AND THEN THE 95, 90, AND 80 PERCENT    *
C     * LOWER CONFIDENCE LIMITS ARE PICKED.  THIS ESTABLISHES THE 95,    *
C     * 90, AND 80 PERCENT CONFIDENCE INTERVALS FOR EACH SYSTEM AND IT   *
C     * IS NOTED WHETHER EACH OF THESE INTERVALS CONTAINS THE TRUE        *
C     * SYSTEM RELIABILITY.                                              *
C     * THE ABOVE PROCESS IS REPEATED FOR NOLMC MONTE CARLO RUNS, WITH   *
C     * COUNTERS FOR EACH SYSTEM TO TRACK THE NUMBER OF TIMES THAT THE   *
C     * THE ABOVE PROCESS IS REPEATED FOR NOLMC MONTE CARLO RUNS, WITH   *
C     * COUNTERS FOR EACH SYSTEM TO TRACK THE NUMBER OF TIMES THAT THE   *
C     * CONFIDENCE INTERVALS CONTAIN THE TRUE SYSTEM RELIABILITY.        *
C     *                                                                  *
C     ********************************************************************
C
      DIMENSION   ARCHAT (2000,5), ATRS (4), BI (12,15), C (4,4), DEV
     1   (2000), PARAM (3,5), QNTKS (40), R (205,5), RC (2000,5), RCHAT
     2   (5), RCHSIG (5), RLBS (12), RS (2000), SIG (12,15), SIGSQ
     3   (12,15), SMSZ (15), TEMP (200), TRC (5), TRS (1), VRH (10,11),
     4   WK (200), WKAREA (10), X (2)
      EXTERNAL    F
      DOUBLE      PRECISIONDSEED
      REAL        KHAT
      DATA    RLBS / .5, .55, .6, .65, .7, .75, .8, .85, .9, .925, .95,
     1   .98 /
C
      DATA    SMSZ / 8., 9., 10., 11., 12., 13., 14., 15., 20., 25.,
     1   30., 40., 50., 75., 100. /
C
```

```
      DATA    VRH / 266., 242., 194., 163., 130., 95., 59., 41., 25.,
     1   6., 200., 187., 153., 130., 103., 76., 47., 33., 19., 5., 167.,
     2   154., 126., 107., 86., 63., 39., 27., 16., 4., 124., 118., 99.,
     3   86., 70., 51., 32., 22., 13., 3., 90., 86., 72., 62., 51., 37.,
     4   23., 16., 9., 2., 72., 68., 58., 50., 41., 30., 19., 13., 7.,
     5   2., 59., 57., 48., 42., 34., 25., 16., 11., 6., 1., 43., 42.,
     6   36., 31., 26., 19., 12., 8., 5., 1., 34., 33., 29., 25., 20.,
     7   15., 9., 7., 4., 1., 23., 22., 19., 17., 14., 10., 6., 4., 3.,
     8   1., 17., 16., 14., 12., 10., 8., 5., 3., 2., 1. /
C
      DATA    QNTKS / .975, .842, .708, .624, .563, .519, .483, .454,
     1   .430, .409, .391, .375, .361, .349, .338, .327, .318, .309,
     2   .301, .294, .287, .281, .275, .269, .264, .259, .254, .250,
     3   .246, .242, .238, .234, .231, .227, .224, .221, .218, .215,
     4   .213, .210 /
C
C  INITIALIZE VARIABLES
      PI        = 4. * ATAN(1.)
      DSEED     = 4000.0D0
      T         = 32.
      CALL RANSET (T)
      TIME      = 100.
      NRLBS     = 12
      NSMSZ     = 15
      NSMSZ1    = NSMSZ - 1
      NRLBS1    = NRLBS - 1
      ISD       = 12
      NTRUNC    = 0
      NS1C99    = 0
      NS1C95    = 0
      NS1C90    = 0
      NS1C80    = 0
      NS1C70    = 0
      NS1C60    = 0
      NS1C50    = 0
      NS2C99    = 0
      NS2C95    = 0
      NS2C90    = 0
      NS2C80    = 0
      NS2C70    = 0
      NS2C60    = 0
      NS2C50    = 0
      NS3C99    = 0
      NS3C95    = 0
      NS3C90    = 0
      NS3C80    = 0
      NS3C70    = 0
      NS3C60    = 0
      NS3C50    = 0
      NS4C99    = 0
      NS4C95    = 0
      NS4C90    = 0
      NS4C80    = 0
      NS4C70    = 0
```

```
      NS4C60    = 0
      NS4C50    = 0
C
      PRINT  409
C  READ THE SAMPLE SIZE AND NUMBER OF MONTE CARLO RUNS
      READ *, NSAM, NOLMC
      PRINT  419, NSAM, NOLMC
      RNSAM     = NSAM
C  READ THE TRUE COMPONENT PARAMETERS
      READ *, ((PARAM(I, J), I=1, 3), J=1, 5)
      PRINT  429, ((J, (PARAM(I, J), I=1, 3), CMPREL(TIME, PARAM(1, J),
     1   PARAM(2, J), PARAM(3, J))), J=1, 5)
      READ (1, *)   ((SIG(I, J), J=1, 15), I=1, 12)
      READ (1, *)   ((BI(I, J), J=1, 15), I=1, 12)
        DO 10 I   = 1, 12
          DO 10 J   = 1, 15
          SIGSQ(I, J) = SIG(I, J) * SIG(I, J) * 1.E4
10        CONTINUE
      PRINT  439, ((SIG(I, J), J=1, 15), I=1, 12)
      PRINT  439, ((BI(I, J), J=1, 15), I=1, 12)
C
C   ***********************************************************************
C   ***********************************************************************
C  THIS OUTSIDE LOOP FROM HERE TO STATEMENT 300 COMPLETES
C  NOLMC MONTE CARLO RUNS OF THE SIMULATION.
C
        DO 300 NCOUNT = 1, NOLMC
C
C   ***********************************************************************
C
C  FOR EACH OF 5 COMPONENTS, THIS LOOP GENERATES THE MLE OF RCHAT
C  FROM THE MLES OF K AND THETA.  IT DETERMINES THE VARIANCE OF
C  RCHAT AND THEN SAMPLES FROM THE NORMAL DISTRIBUTION FOR NDEV
C  SAMPLE RELIABILITIES OF EACH COMPONENT.
C
          DO 290 J  = 1, 5
          TK        = PARAM(1, J)
          TTHETA    = PARAM(2, J)
          TC        = PARAM(3, J)
C  PASS INFORMATION ON COMPONENT NUMBER AND
C  SAMPLE SIZE THRU ARRAY R
          R(205, 1) = J
          R(204, J) = NSAM
C  DETERMINE THE TRUE COMPONENT RELIABILITY
          TRC(J)    = CMPREL(TIME, TK, TTHETA, TC)
C
C  GATHER NSAM FAILURE TIMES FROM THE
C  WEIBULL DISTRIBUTION WITH TRUE PARAMETERS
C  TK,TTHETA, AND TC
C
          CALL GGWIB (DSEED, TK, NSAM, TEMP)
            DO 20 I   = 1, NSAM
            R(I, J)   = TTHETA * TEMP(I) + TC
20          CONTINUE
```

```
C  DETERMINE THE LIKELIHOOD OF DRAWING THIS SAMPLE
C  USING THE TRUE K, THETA, AND C
C
C  DETERMINE THE MLE OF THETA AND K
C  BY HARTER & MOORES ITERATIVE SCHEME
C  (ZSYSTM SOLVES THE TWO NONLINEAR SIMULTANEOUS EQUATIONS)
C
           EPS       = 1.E - 10
           NSIG      = 8
           N         = 2
           NSTART    = 0
           START     = 2.55
   30      ITMAX     = 50
           SUMXIK    = 0.
           X(1)      = START
             DO 40 I   = 1, NSAM
   40        SUMXIK    = SUMXIK + R(I, J) ** X(1)
           X(2)      = (SUMXIK / NSAM) ** (1. / X(1))
           CALL ZSYSTM (F, EPS, NSIG, N, X, ITMAX, WKAREA, R, IER)
           KHAT      = X(1)
           THAT      = X(2)
C
           IF (ITMAX .EQ. 50 .OR. KHAT .LT. 0. .OR. THAT .LT. 0. .OR.
     1       IER.NE.0) GO TO 200
C     AND PRINT ERROR MESSAGE THEN RESTART ITERATION
C     WITH A DIFFERENT KHAT
C
C  DETERMINE RHAT, THE MLE OF THE COMPONENT RELIABILITY, USING
C  THE TRUE C AND THE MLES OF K AND THETA
           RCHAT(J)  = CMPREL(TIME, KHAT, THAT, TC)
C
C  GIVEN THE MLE OF THE COMPONENT RELIABILITY AND THE SAMPLE SIZE,
C  ENTER THE 2-DIMENSIONAL ARRAY BI AND FIND THE BIAS OF THE
C  ESTIMATOR
             DO 50 I   = 1, NSMSZ1
             LSMSZ     = I
             IF (RNSAM .LE. SMSZ(I+1)) GO TO 60
   50        CONTINUE
           GO TO 90
   60      IF (RCHAT(J) .LT. .5) GO TO 100
             DO 70 I   = 1, NRLBS1
             LRLBS     = I
             IF (RCHAT(J) .LE. RLBS(I+1)) GO TO 80
   70        CONTINUE
           GO TO 90
   80      CALL IBCICU (BI, ISD, RLBS, NRLBS, SMSZ, NSMSZ, LRLBS, LSMSZ,
     1       C, WK, IER)
           CALL IBCEVU (RLBS, NRLBS, SMSZ, NSMSZ, LRLBS, LSMSZ, C,
     1       RCHAT(J), RNSAM, BIAS, IER)
           IF (LRLBS .LT. NRLBS) GO TO 110
   90      BIAS      = 0.
           GO TO 110
  100      IF  (RNSAM .EQ. SMSZ(LSMSZ + 1)) LSMSZ = LSMSZ + 1
           BIAS      = BI(1, LSMSZ)
```

```
  110       RCHAT(J)  = RCHAT(J) - BIAS
            ARCHAT(NCOUNT, J) = RCHAT(J)
C
C  GIVEN THE UNBIASED ESTIMATOR OF THE COMPONENT RELIABILITY,
C  ENTER THE 2-DIMENSIONAL ARRAY SIG AND FIND THE STANDARD
C  DEVIATION OF THE UNBIASED ESTIMATOR
            Z         = RCHAT(J)
            RCRLB     = SQRT(Z ** 2 * (ALOG(Z)) ** 2 * (1.109 - .514 *
     1        ALOG( - ALOG(Z)) + .608 * (ALOG( - ALOG(Z))) ** 2) / RNSAM)
              DO 120 I  = 1, NSMSZ1
              LSMSZ     = I
              IF (RNSAM .LE. SMSZ(I+1)) GO TO 130
  120         CONTINUE
            GO TO 160
  130       IF (RCHAT(J) .LT. .5) GO TO 170
  130       IF (RCHAT(J) .LT. .5) GO TO 170
              DO 140 I  = 1, NRLBS1
              LRLBS     = I
              IF (RCHAT(J) .LE. RLBS(I+1)) GO TO 150
  140         CONTINUE
            GO TO 160
  150       CALL IBCICU (SIG, ISD, RLBS, NRLBS, SMSZ, NSMSZ, LRLBS,
     1        LSMSZ, C, WK, IER)
            CALL IBCEVU (RLBS, NRLBS, SMSZ, NSMSZ, LRLBS, LSMSZ, C,
     1        RCHAT(J), RNSAM, SIGMA, IER)
            IF (SIGMA .GE. RCRLB) GO TO 180
  160       SIGMA     = RCRLB
            IER       = 0
            GO TO 180
  170       IF  (RNSAM .EQ. SMSZ(LSMSZ + 1)) LSMSZ = LSMSZ + 1
            SIGMA     = SIG(1, LSMSZ)
            IF (SIGMA .GE. RCRLB) GO TO 180
            SIGMA     = RCRLB
  180       CONTINUE
C
C  FORM A VECTOR OF NDEV SAMPLE RELIABILITIES WITH MEAN RCHAT
C  AND A STANDARD DEVIATION OF SIGMA
            NDEV      = 2000
            CALL GGNML (DSEED, NDEV, DEV)
              DO 190 I  = 1, 2000
              RC(I, J)  = RCHAT(J) + DEV(I) * SIGMA
C  TRUNCATE THE NORMAL DISTRIBUTION IF COMPONENT RELIABILITY
C  IS GREATER THAN 1
              IF (RC(I,J) .LE. 1.) GO TO 190
              RC(I, J)  = 1.
              NTRUNC    = NTRUNC + 1
  190         CONTINUE
            GO TO 290
C
C  IF FAILS TO CONVERGE, START WITH ANOTHER KHAT AND TRY AGAIN
  200       PRINT  449, NCOUNT, J, NSTART, KHAT, THAT, ITMAX, IER
            NSTART    = NSTART + 1
            GO TO  ( 210, 220, 230, 240, 250, 260, 270, 280),  NSTART
  210       START     = 1.
```

```
            GO TO 30
  220       START     = .5
            GO TO 30
  230       START     = .9
            GO TO 30
  240       START     = 4.5
            GO TO 30
  250       START     = 1.1
            GO TO 30
  260       START     = 1.5
            GO TO 30
  270       START     = 3.5
            GO TO 30
C  IF MLES FAIL TO CONVERGE IN 8 ATTEMPTS, PRINT A
C  MESSAGE AND GO ON TO THE NEXT COMPONENT
  280       PRINT  459
C
  290       CONTINUE
C  ************************************************************************
C
C  FOR EACH OF THE 4 SYSTEMS, THE DIFFERENT COMPONENTS ARE COMBINED
C  TO YIELD NDEV SAMPLES OF THE SYSTEM RELIABILITY.  THESE SAMPLES
C  ARE SEQUENCED IN ASCENDING ORDER AND THEN COUNTERS KEEP TRACK
C  OF WHEN THE 99, 95, 90, 80, 70, 60, AND 50 PERCENT CONFIDENCE
C  INTERVALS CONTAIN THE TRUE SYSTEM RELIABILITY.
C
C  SYSTEM 1
          NMC       = 1
          IDIM      = 1
          CALL REL1 (NMC, TRC, TRS, IDIM)
          ATRS(1)   = TRS(1)
          NMC       = 2000
          IDIM      = 2000
          CALL REL1 (NMC, RC, RS, IDIM)
          CALL VSRTA (RS, IDIM)
          IF  (RS(20) .LE. TRS(1)) NS1C99 = NS1C99 + 1
          IF  (RS(100) .LE. TRS(1)) NS1C95 = NS1C95 + 1
          IF  (RS(200) .LE. TRS(1)) NS1C90 = NS1C90 + 1
          IF  (RS(400) .LE. TRS(1)) NS1C80 = NS1C80 + 1
          IF  (RS(600) .LE. TRS(1)) NS1C70 = NS1C70 + 1
          IF  (RS(800) .LE. TRS(1)) NS1C60 = NS1C60 + 1
          IF  (RS(1000) .LE. TRS(1)) NS1C50 = NS1C50 + 1
C
C  SYSTEM 2
          NMC       = 1
          IDIM      = 1
          CALL REL2 (NMC, TRC, TRS, IDIM)
          ATRS(2)   = TRS(1)
          NMC       = 2000
          IDIM      = 2000
          CALL REL2 (NMC, RC, RS, IDIM)
          CALL VSRTA (RS, IDIM)
          IF  (RS(20) .LE. TRS(1)) NS2C99 = NS2C99 + 1
          IF  (RS(100) .LE. TRS(1)) NS2C95 = NS2C95 + 1
```

```
          IF  (RS(200) .LE. TRS(1)) NS2C90 = NS2C90 + 1
          IF  (RS(400) .LE. TRS(1)) NS2C80 = NS2C80 + 1
          IF  (RS(600) .LE. TRS(1)) NS2C70 = NS2C70 + 1
          IF  (RS(800) .LE. TRS(1)) NS2C60 = NS2C60 + 1
          IF  (RS(1000) .LE. TRS(1)) NS2C50 = NS2C50 + 1
C
C  SYSTEM 3
          NMC       = 1
          IDIM      = 1
          CALL REL3 (NMC, TRC, TRS, IDIM)
          ATRS(3)   = TRS(1)
          NMC       = 2000
          IDIM      = 2000
          CALL REL3 (NMC, RC, RS, IDIM)
          CALL VSRTA (RS, IDIM)
          IF  (RS(20) .LE. TRS(1)) NS3C99 = NS3C99 + 1
          IF  (RS(100) .LE. TRS(1)) NS3C95 = NS3C95 + 1
          IF  (RS(200) .LE. TRS(1)) NS3C90 = NS3C90 + 1
          IF  (RS(400) .LE. TRS(1)) NS3C80 = NS3C80 + 1
          IF  (RS(600) .LE. TRS(1)) NS3C70 = NS3C70 + 1
          IF  (RS(800) .LE. TRS(1)) NS3C60 = NS3C60 + 1
          IF  (RS(1000) .LE. TRS(1)) NS3C50 = NS3C50 + 1
C
C  SYSTEM 4
          NMC       = 1
          IDIM      = 1
          CALL REL4 (NMC, TRC, TRS, IDIM)
          ATRS(4)   = TRS(1)
          NMC       = 2000
          IDIM      = 2000
          CALL REL4 (NMC, RC, RS, IDIM)
          CALL VSRTA (RS, IDIM)
          IF  (RS(20) .LE. TRS(1)) NS4C99 = NS4C99 + 1
          IF  (RS(100) .LE. TRS(1)) NS4C95 = NS4C95 + 1
          IF  (RS(200) .LE. TRS(1)) NS4C90 = NS4C90 + 1
          IF  (RS(400) .LE. TRS(1)) NS4C80 = NS4C80 + 1
          IF  (RS(600) .LE. TRS(1)) NS4C70 = NS4C70 + 1
          IF  (RS(800) .LE. TRS(1)) NS4C60 = NS4C60 + 1
          IF  (RS(1000) .LE. TRS(1)) NS4C50 = NS4C50 + 1
  300     CONTINUE
C
C   ***********************************************************************
C   ***********************************************************************
C
        NPT       = 10000 * NOLMC
        PRINT  469, NTRUNC, NPT
        RNOLMC    = NOLMC
C  FOR SYSTEM 1, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
        PS1C99    = NS1C99 / RNOLMC
        PS1C95    = NS1C95 / RNOLMC
        PS1C90    = NS1C90 / RNOLMC
        PS1C80    = NS1C80 / RNOLMC
        PS1C70    = NS1C70 / RNOLMC
```

```
      PS1C60    = NS1C60 / RNOLMC
      PS1C50    = NS1C50 / RNOLMC
C  FOR SYSTEM 2, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
      PS2C99    = NS2C99 / RNOLMC
      PS2C95    = NS2C95 / RNOLMC
      PS2C90    = NS2C90 / RNOLMC
      PS2C80    = NS2C80 / RNOLMC
      PS2C70    = NS2C70 / RNOLMC
      PS2C60    = NS2C60 / RNOLMC
      PS2C50    = NS2C50 / RNOLMC
C  FOR SYSTEM 3, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
      PS3C99    = NS3C99 / RNOLMC
      PS3C95    = NS3C95 / RNOLMC
      PS3C90    = NS3C90 / RNOLMC
      PS3C80    = NS3C80 / RNOLMC
      PS3C70    = NS3C70 / RNOLMC
      PS3C60    = NS3C60 / RNOLMC
      PS3C50    = NS3C50 / RNOLMC
C  FOR SYSTEM 4, DETERMINE THE 99, 95, 90, 80, 70, 60, AND 50
C  PERCENT CONFIDENCE LIMIT COVERAGE OF THE TRUE SYSTEM RELIABILITY
      PS4C99    = NS4C99 / RNOLMC
      PS4C95    = NS4C95 / RNOLMC
      PS4C90    = NS4C90 / RNOLMC
      PS4C80    = NS4C80 / RNOLMC
      PS4C70    = NS4C70 / RNOLMC
      PS4C60    = NS4C60 / RNOLMC
      PS4C50    = NS4C50 / RNOLMC
      PRINT  479
      PRINT  489, ATRS (1), PS1C99, PS1C95, PS1C90, PS1C80, PS1C70,
     1   PS1C60, PS1C50
      PRINT  499
      PRINT  489, ATRS (2), PS2C99, PS2C95, PS2C90, PS2C80, PS2C70,
     1   PS2C60, PS2C50
      PRINT  509
      PRINT  489, ATRS (3), PS3C99, PS3C95, PS3C90, PS3C80, PS3C70,
     1   PS3C60, PS3C50
      PRINT  519
      PRINT  489, ATRS (4), PS4C99, PS4C95, PS4C90, PS4C80, PS4C70,
     1   PS4C60, PS4C50
C
C   ***********************************************************************
C   ***********************************************************************
C
C   THIS PROGRAM SEGMENT RUNS A KOLMOGOROV-SMIRNOV TEST OF THE
C   ASSUMPTION THAT THE COMPONENT RHATS ARE NORMALLY DISTRIBUTED
C
      DO 360 J  = 1, 5
      Z         = TRC(J)
      RCRLB     = SQRT(Z ** 2 * (ALOG(Z)) ** 2 * (1.109 - .514 *
     1   ALOG( - ALOG(Z)) + .608 * (ALOG( - ALOG(Z))) ** 2) / RNSAM)
        DO 310 I  = 1, NSMSZ1
        LSMSZ     = I
```

```
          IF (RNSAM .LE. SMSZ(I+1)) GO TO 320
  310     CONTINUE
          GO TO 350
  320     DO 330 I  = 1, NRLBS1
          LRLBS     = I
          IF (TRC(J) .LE. RLBS(I+1)) GO TO 340
  330     CONTINUE
          GO TO 350
  340     CALL IBCICU (SIG, ISD, RLBS, NRLBS, SMSZ, NSMSZ, LRLBS, LSMSZ,
     1      C, WK, IER)
          CALL IBCEVU (RLBS, NRLBS, SMSZ, NSMSZ, LRLBS, LSMSZ, C, TRC(J),
     1      RNSAM, SIGMA, IER)
          IF (SIGMA .GE. RCRLB) GO TO 360
  350     SIGMA     = RCRLB
          IER       = 0
          PRINT *, "Z=", Z, "   SIGMA=", SIGMA
  360     RCHSIG(J) = SIGMA
        PRINT  529, (TRC(J), J=1, 5)
        PRINT  529, (RCHSIG(J), J=1, 5)
        PRINT *, " "
        ALPHA     = .05
        TALPHA    = 1.36 / SQRT(RNOLMC)
        IF  (NOLMC .LE. 40) TALPHA = QNTKS (NOLMC)
        CALL TNORKS (ARCHAT, NOLMC, TRC, RCHSIG, ALPHA, TALPHA)
C
C   ***************************************************************************
C   ***************************************************************************
C
        STOP "FORAWHILE"
C

  409   FORMAT  ( 1H1  )
  419   FORMAT  ( " ********************************************************"
     1    "**********",  / ,  " *", T62,  "*",  / ,  " *", T62,  "*",  /
     2    ,  " *", T25,  "SAMPLE "  "SIZE = ", I3, T62,  "*",  / ,  " *",
     3    T62,  "*",  / ,  " *", T22,  "MONTE CARLO "  "SIZE = ", I3,
     4    T62,  "*",  / ,  " *", T62,  "*",  / ,  " *", T62,  "*",  / ,
     5    " ************************************************************"
     6    "*",  /  /  /  /  /  /  /  )
  429   FORMAT  ( 5(1X,  "COMPONENT ", I1,  / , 6X,  "K = ", F4.2,  / ,
     1    6X,  "THETA = ", F5.0,  / , 6X,  "C = ", F2.0,  / , 6X,
     2    "RELIABILITY =", F7.5,  /  / ) )
  439   FORMAT  ( 15(1X, F7.5) )
  449   FORMAT  ( / , " **********",  / ,  " MC=", I4, 4X,  "J =", I2,
     1    4X,  "NSTART =", I2,  / ,  " KHAT=" E13.6, 5X "THAT=" E13.6 /
     2    " ITERATIONS=" I3, 5X "IER=" I3 /  " **********"  )
  459   FORMAT  ( /  " ********************" /  /  " DID NOT CONVERGE IN"
     1    " 8 ATTEMPTS  WITH DIFFERENT STARTING KHATS" /  " THEREFORE GO"
     2    "ING ON TO THE NEXT   COMPONENT" /  /  " ********************"
     3    )
  469   FORMAT  ( /  / ,  " THERE WERE", I7,  " HIGH TRUNCATIONS OUT OF"
     1    , I8,  " RELIABILITY DEVIATES",  /  /  )
  479   FORMAT  ( /  " ***** SYSTEM 1 *****" /  " (3 COMPONENTS IN SERI"
     1    "ES)"  )
```

```
  489  FORMAT  (  / ,  " TRUE SYSTEM RELIABILITY =", F7.5,  /
     1   " THE 99 PERCENT CONFIDENCE INTERVAL COVERED ", F6.4,
     2   " OF THE RUNS",  / ,  " THE 95 PERCENT CONFIDENCE INTERVAL COV"
     3   "ERED ", F6.4 "OFTHERUNS",  / ,  " THE 90 PERCENT CONFIDENCE I"
     4   "NTERVAL COVERED " F6.4,  " OF THE RUNS",  / ,  " THE 80 PERCE"
     5   "NT CONFIDENCE INTERVAL"  " COVERED " F6.4 "OFTHERUNS",  / ,
     6   " THE 70 PERCENT CONFIDENCE "  "INTERVAL COVERED ", F6.4,
     7   " OF THE RUNS",  / ,  " THE 60 PERCENT "  "CONFIDENCE INTERVAL"
     8   " COVERED ", F6.4,  " OF THE RUNS",  / ,  " THE",  " 50 PERCEN"
     9   "T CONFIDENCE INTERVAL COVERED ", F6.4,  " OF THE RUNS",  /  /
     9   /  )
  499  FORMAT  (  /  " ***** SYSTEM 2 *****" /  " (1 COMPONENT IN SERIE"
     1   "S WITH 2 "  "IN PARALLEL)"  )
  509  FORMAT  (  /  " ***** SYSTEM 3 *****" /  " (3 COMPONENTS IN PARA"
     1   "LLEL)"  )
  519  FORMAT  (  /  " ***** SYSTEM 4 *****" /  " (A 5-COMPONENT COMPLE"
     1   "X NETWORK)"  )
  529  FORMAT  ( 5(1X, F10.8, 5X) )
C
       END
```

```
FUNCTION  CMPREL (TIME, K, THETA, C)
REAL        K
CMPREL    = EXP( - ((TIME - C) / THETA) ** K)
RETURN
END
```

```
      FUNCTION  RLKLHD (R, RK, T, C)
      DIMENSION   R (205,5)
      J         = R(205, 1)
      NSAM      = R(204, J)
      RLKLHD    = 1.
        DO 10 I   = 1, NSAM
        X         = RLKLHD
10      RLKLHD    = X * RK * (R(I, J) - C) ** (RK - 1.) / T ** RK *
     1    EXP( - (((R(I, J) - C) / T) ** RK))
      RETURN
      END
```

```
      FUNCTION  F (X, K, R)
      DIMENSION   R (205,5), X (2)
      REAL        KHAT
      J         = R(205, 1)
      NSAM      = R(204, J)
      KHAT      = X(1)
      THAT      = X(2)
      SUMXIK    = 0.
      SUMLXI    = 0.
      SUMFR     = 0.
      IF (THAT .LE. 0. .OR. KHAT .GT. 15.) RETURN
        DO 10 I   = 1, NSAM
        SUMXIK    = SUMXIK + R(I, J) ** KHAT
10      SUMLXI    = SUMLXI + ALOG(R(I, J))
        DO 20 I   = 1, NSAM
20      SUMFR     = SUMFR + (R(I, J) / THAT) ** KHAT * ALOG(R(I, J) /
     1    THAT)
      IF (K .EQ. 2) GO TO 30
      F         = THAT - (SUMXIK / NSAM) ** (1. / KHAT)
      RETURN
30    F         = NSAM / KHAT - NSAM * ALOG(THAT) + SUMLXI - SUMFR
      RETURN
      END
```

```
      SUBROUTINE  REL1 (NMC, R, RS, IDIM)
C  REL1 DETERMINES THE SYSTEM RELIABILITY OF 3 COMPONENTS IN SERIES
      DIMENSION   R (IDIM,5), RS (IDIM)
        DO 10 I   = 1, NMC
   10   RS(I)     = R(I, 1) * R(I, 2) * R(I, 3)
      RETURN
      END
```

```
      SUBROUTINE  REL2 (NMC, R, RS, IDIM)
C  REL2 DETERMINES THE SYSTEM RELIABILITY OF 1 COMPONENT IN
C  SERIES WITH 2 IN PARALLEL
      DIMENSION   R (IDIM,5), RS (IDIM)
        DO 10 I   = 1, NMC
   10   RS(I)     = R(I, 1) * (1. - (1. - R(I, 2)) * (1. - R(I, 3)))
      RETURN
      END
```

```
      SUBROUTINE  REL3 (NMC, R, RS, IDIM)
C  REL3 DETERMINES THE SYSTEM RELIABILITY OF 3 COMPONENTS IN PARALLEL
      DIMENSION   R (IDIM,5), RS (IDIM)
        DO 10 I   = 1, NMC
   10   RS(I)     = 1. - (1. - R(I, 1)) * (1. - R(I, 2)) * (1. - R(I,
     1     3))
      RETURN
      END
```

```
      SUBROUTINE  REL4 (NMC, R, RS, IDIM)
C  REL4 DETERMINES THE SYSTEM RELIABILITY OF A 5 COMPONENT
C  COMPLEX NETWORK
      DIMENSION   R (IDIM,5), RS (IDIM)
        DO 10 I   = 1, NMC
        RS(I)     = R(I, 1) * (1 - (1 - R(I, 2)) * (1 - R(I, 5) * (1 -
     1    (1 - R(I, 3)) * (1 - R(I, 4)))))
   10   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE  TNORKS (ARCHAT, NOLMC, TRC, RCHSIG, ALPHA, TALPHA)
      DIMENSION   ARCHAT (2000,5), CHMX (5), RCHSIG (5), TRC (5)
      RNOLMC    = NOLMC
C  ORDER EACH COLUMN (A VECTOR CONTAINING THE COMPONENT RHATS)
C  FROM THE SMALLEST RHAT TO THE LARGEST
        DO 10 J   = 1, 5
   10   CALL VSRTA (ARCHAT(1, J), NOLMC)
C
C  ********** FOR EACH COMPONENT **********
        DO 40 J   = 1, 5
C
C  FIND THE DEVIATIONS OF THE EMPERICAL DISTRIBUTION FROM THE
C  CUMULATIVE PROBABILITY DISTRIBUTION OF THE NORMAL
        AVRCH     = 0.
          DO 20 I   = 1, NOLMC
C  AVRCH SUMS THE RHATS FOR EACH COMPONENT.  LATER IT WILL BE
C  DIVIDED BY THE MONTE CARLO SIZE TO GET AN AVERAGE RHAT.
          AVRCH     = AVRCH + ARCHAT(I, J)
          SD        = (ARCHAT(I, J) - TRC(J)) / RCHSIG(J)
          CALL MDNOR (SD, X)
          Z         = I
          CHLO      = ABS((Z - 1.) / RNOLMC - X)
          CHHI      = ABS(Z / RNOLMC - X)
          IF  (I .EQ. 1) CHMX (J) = CHLO
          IF  (CHLO .GT. CHMX(J)) CHMX (J) = CHLO
          IF  (CHHI .GT. CHMX(J)) CHMX (J) = CHHI
          PRINT *, J, I, ARCHAT (I, J), X, CHLO, CHHI
   20     CONTINUE
C  PRINT THE RESULTS OF THE K-S TEST OF NORMALITY
        AVRCH     = AVRCH / RNOLMC
        BIAS      = AVRCH - TRC(J)
        ESD       = 0.
          DO 30 I   = 1, NOLMC
          DIFF      = ARCHAT(I, J) - AVRCH
          ESD       = ESD + DIFF * DIFF
   30     CONTINUE
        ESD       = SQRT(ESD / (RNOLMC - 1.))
        PRINT  109, J, TRC (J), RCHSIG (J), AVRCH, ESD, BIAS, CHMX (J),
     1    TALPHA
        IF (CHMX(J) .GT. TALPHA) PRINT  119,ALPHA
        IF (CHMX(J) .LE. TALPHA) PRINT  129,ALPHA
   40   CONTINUE
C  ****************************************
C
      RETURN
C
C
  109  FORMAT  (  / , 1X,  "K-S TEST OF NORMALITY FOR RHATS OF COMPONEN"
     1   "T ", I1,  / , 5X,  "TRUE COMPONENT RELIABILITY =", F10.8,  / ,
     2   5X,  "SIGMA =", F10.8,  / , 5X,  "AVERAGE RHAT =", F10.8,  / ,
     3   5X,  "EMPERICAL STANDARD DEVIATION =", F10.8,  / , 5X,
     4   "BIAS =", F10.8,  / , 5X,  "TEST STATISTIC =", F5.3, 8X,
     5   "TALPHA =", F5.3 )
  119  FORMAT  ( 5X,  "REJECT AT ALPHA =", F4.2 )
```

```
  129  FORMAT  ( 5X,  "FAIL TO REJECT AT ALPHA =", F4.2 )
C
       END
```

Vita

Randall Blanton Putz, the son of Leo F. and Marjorie Putz, was born on 13 March 1951, in Corpus Christi, Texas. After graduating from W. B. Ray High School in Corpus Christi, he went to the United States Air Force Academy, where he received his bachelor's degree in Engineering Sciences in 1973 and was commissioned a 2nd Lieutenant in the United States Air Force. After attending pilot training, he received his wings at Laughlin Air Force Base, Texas, in 1974, where he served as an instructor pilot until coming to the Air Force Institute of Technology in June 1978. He is married to the former Nancy Walker of San Antonio, Texas.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A univariate Monte Carlo technique is developed for the determination of lower confidence limits of system reliability based on component test data. It is assumed that the component test data consists of failure times which are distributed according to a known two-parameter Weibull probability distribution. These failure times are randomly generated using the true shape and scale parameters of the distribution. Maximum-likelihood

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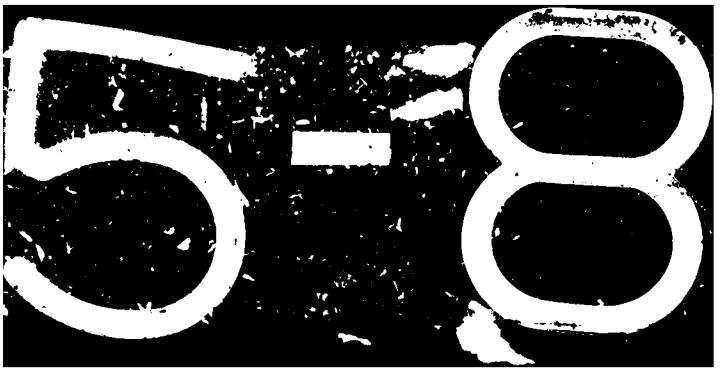
estimators are found for the shape and scale parameters and then substituted into the reliability equation to obtain the maximum-likelihood estimator for the component reliability. The estimated bias in this estimator is subtracted to yield an approximately unbiased estimator of the component reliability. Using the empirical variance of the reliability estimate and assuming a normal distribution, a Monte Carlo simulation is run for four hypothetical systems consisting of as many as five components. The simulation is repeated 600 times. Since the true reliability is known, on each run it can be determined if the desired confidence intervals contain the true system reliability. The result is an absolute measure of the effectiveness of the univariate technique. The entire simulation was run for component test data sample sizes ranging from ten to one hundred. A second run of 600 was made to examine the Monte Carlo sampling error for component test sample sizes of 100.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